PROGRAM + ABSTRACTS

TAPDE 2026

Tbilisi Analysis & PDE Workshop

IN MEMORIAM OF

Nikoloz Muskhelishvili

(1891-1976)

AUGUST 26-29, 2026

V. KUPRADZE INSTITUTE OF MATHEMATICS

University of Georgia, Tbilisi

The book of abstracts has been assembled by **Maia Japoshvili**
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Contents

About

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TAPDE 2026

The Tbilisi Analysis & PDE Seminar (TAPDE) was established in November 2020 as a bi-weekly series of online seminars, featuring presentations from over 70 distinguished scholars from various countries. Although the initial plan included complementing the online talks with an annual workshop, this was postponed due to the COVID-19 pandemic until 2023, when the first **Tbilisi Analysis & PDE Workshop** (TAPDE 2023)[1] was held in Tbilisi, the capital of Georgia. This workshop has now become an annual international event.

The workshop covers a broad range of topics, including Real and Complex analysis, Operator Theory, Harmonic Analysis, Integral Equations, Numerical Analysis, Partial Differential Equations, Mathematical Physics, and related fields. We invite researchers to submit their work for presentation, share their latest research findings, and engage in discussions about new developments in their respective areas of expertise.

Committees

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Editors: Roland Duduchava, Eugene Shargorodsky, Emzar Khmaladze

[1] https://tapde-workshop.ug.edu.ge/2023/

Agenda

Invited Talks (IT): The invited talks are scheduled to last **60 minutes**, with **5 minutes** included for questions.
Contributed Talks (CT): The contributed talks are scheduled for **30 minutes**, including **5 minutes** for questions.

August 26, Wednesday, Session at the University of Georgia, Tbilisi

09:00 – 09:30	**Registration at the entrance of Room 519**	
09:30–10:00	**Opening, Room 519**	
Room 519	**Plenary Session** **Chairman – Roland Duduchava**	
10:00 – 10:50	**Alex Iosevich** (New York, USA) – Spectral Synthesis and Complexity	**IT**
Room 315	**Plenary Session** **Chairman – Ilya Spitkovski**	
10:00 – 10:50	**Volker John** (Berlin, Germany) – A Class of inf-sup Slized Scott-Vogelius Pairs on General Shape-Regular Simplicial Grids	**IT**
11:30 – 12:20	**Coffee Break, Room 519**	
Room 519	**Plenary Session** **Chairman – Roland Duduchava**	
11:30 – 12:20	**Durvudkhan Suragan** (Astana, Kazakhstan) – Geometric Hardy Inequalities on the Heisenberg Group	**IT**
Room 305	**Plenary Session** **Chairman – Ilya Spitkovski**	
11:30 – 12:20	**Malte Braack** (Kiel, Germany), Hannah Scherer – Mapped Coercivity for Real- and Complex-Valued PDEs	**IT**
12:30 – 14:30	**Lunch, Room 519**	
Room 519	**Plenary Session** **Chairman – Ryan O'Loughlin**	
14:30 – 15:20	Javier Carrillo-Alanís, **Guillermo P. Curbera** (Sevilla, Spain), Olvido Delgado – On the Söhngen Problem for the Finite Hilbert Transform	**IT**
Room 305	**Plenary Session** **Chairman – Sergey Tikonov**	
14:30 – 15:20	**Dmitri Kuzmin** (Dortmund, Germany) – Element-Based WENO Stabilization and Convex Limiting for Finite Element Discretizations of Hyperbolic Problems	**IT**
15:30 – 16:00	**Coffee Break, Room 519**	

Room 305	**Session: Partial Differential Equations** **Chairman – Dmitry Kuzmin**	
16:00 – 16:30	**Zahra Keyshams** (Yerevan, Armenia), Khachatur Khachatryan – Existence and Uniqueness of Solutions to Non-Local Nonlinear Reaction-Diffusion Equations	**CT**
16:30 – 17:00	D. Pilod, J.-C. Saut, S. Selberg, **A. Tesfahun** (Astana, Kazakhstan) – Dispersive Estimates for Full Dispersion KP Equations	**CT**
17:00 – 17:30	**Madina Kamet** (Astana, Kazakhstan), Achenef Tesfahun – Enhanced Long-Time Existence for the Whitham-Boussinesq System with Surface Tension in One and Two Dimensions	**CT**
17:30 – 18:00	A. Khachatryan, **Monire Mikaeili Nia** (Yerevan, Armenia) – Solvability of a System of **Nonlinear** Wave Equations in Three-Dimensional Space	**CT**
Room 519		
18:00	**Welcome Party**	

August 27, Thursday, Session at the University of Georgia, Tbilisi

09:30 – 10:00	**Registration at the entrance of Room 519**	
Room 519	**Plenary Session** **Chairman – Volker John**	
10:00 – 10:50	**Ryan O'Loughlin** (Reading, U.K.) – Bounded Toeplitz Products on the Hardy Space	**IT**
Room 305	**Plenary Session** **Chairman – Carsten Trunk**	
10:00 – 10:50	Nenad Teofanov, **Joachim Toft** (Växjö, Sweden), Patrik Wahlberg – Analytic Pseudo-Differential Calculus Via the Bargmann Transform	**IT**
11:00 – 11:30	**Coffee Break, Room 519**	
Room 519	**Plenary Session** **Chairman – Andreas Rupp**	
11:00 – 12:20	**Elijah Liflyand** (Ramat-Gan, Israel) – Convexity and Fourier Transform	**IT**
Room 305	**Plenary Session** **Chairman – Berikbol T. Torebek**	
11:00 – 12:20	**Luís Castro** (Aveiro, Portugal) – From Convolutions to Polyconvolutions	**IT**
12:30 – 14:30	**Lunch, Room 519**	
Room 519	**Plenary Session** **Chairman – Malte Braack**	
14:30 –15:20	**Alexander Meskhi** (Tbilisi, Georgia) – Olsen's Sharp Inequalities	**IT**

Room 305	**Plenary Session**	
	Chairman – Elijah Liflyand	
14:30 –15:20	**Berikbol Torebek** (Almaty, Kazakhstan) – Critical Curves for Heat Equations with Convolution-Type Nonlinearities	**IT**
15:30 – 16:00	**Coffee Break, Room 519**	
Room 305	**Session: Real and Complex Analysis**	
	Chairman – Alexander Meskhi	
16:00 – 16:30	Vakhtang Tsagareishvili, **Giorgi Tutberidze** (Tbilisi, Georgia) – Some New Sharp Boundedness Result Concerning Linear functionals on Banach Spaces	**CT**
16:30 – 17:00	**Maria Skopina** (Saint Petersburg, Russia) – Orthogonal Wavelet Bases on the Generalized Vilenkin Groups	**CT**
17:00 – 17:30	**Nazife Erkurşun Özcan** (Ankara, Türkiye) – Dobrushin Coefficient and Uniform Ergodicity of Operator Semigroups	**CT**
Room 318	**Session: Partial Differential Equations**	
	Chairman – Duvrudkhan Suragan	
16:00 – 16:30	Jemal Rogava, **Zurab Vashakidze** (Tbilisi, Georgia) – Rational Approximation of Unitary Operator Groups and Applications to Hyperbolic Equations	**CT**
16:30 – 17:00	**Tengiz Buchukuri** (Tbilisi, Georgia), Otar Chkadua, David Natroshvili – Dynamical Mixed Boundary-transmission Problem of Thermo-Electro-Magneto-Elastic Composite body Containing an Interface Crack	**CT**
17:00 – 17:30	Aleksandr Oleinikov, **Tatiana Kuzmina** (Tbilisi, Georgia) – Failure Envelope and Poisson's Effect Equations for Composite Anisogrid Laminates	**CT**

19:00 – Conference dinner at the restaurant “Etnotsiskvili”

August 28, Friday, Excursion to Uplistsikhe and Ateni Sioni Church; Lunch in Gori, Restaurant “Nacharmagevi”

August 29, Sunday, Session at the University of Georgia, Tbilisi

Room 519	**Plenary Session** **Chairman – Alex Iosevich**	
10:00 – 10:50	**Michael Ruzhansky** (Ghent, Belgium) – Noncommutative Fujita Exponent	**IT**
11:00 – 11:30	**Coffee Break, Room 519**	
Room 519	**Plenary Session** **Chairman – Joachim Toft**	
11:30 – 12:20	**Carsten Trunk** (Ilmenau, Germany) – Schrödinger Operator on a Star Graph with Nondefinite Weight Function	**IT**
Room 305	**Plenary Session** **Chairman – Luis Castro**	
11:30 – 12:20	Mukul Dwivedi, Jesse Railo, **Andreas Rupp** (Saarbrücken, Germany) – A Galerkin Finite Element Method for the Fractional Calderón Problem	**IT**
12:30 – 14:30	**Lunch, Room 519**	
Room 519	**Plenary Session** **Chairman – Mikhael Ruzhansky**	
14:30 – 15:20	Miquel Suacedo, **Sergey Tikhonov** (Barcelona, Spain) – Dual of Hardy Inequality	**IT**
Room 305	**Plenary Session** **Chairman – Guilermo Curbera**	
14:30 – 15:20	**Ilya Spitkovsky** (Williamsburg, USA & Abu Dhabi, Emirates) – Numerical Ranges and their Possible Shapes: some Recent Developments	**IT**
Room 519		
15:30	**Closing of the Workshop**	

Abstracts of Invited Talks

Mapped Coercivity for Real- and Complex-Valued PDEs

Malte Braack, Hannah Scherer

Mathematical Deparment, University of Kiel, Germany

We extend the recently introduced theory of mapped coercive operators from real to complex Banach spaces. While classical surjectivity results for nonlinear operators, such as the Browder–Minty theorem, rely on monotonicity in addition to coercivity, mapped coercivity provides a more general framework that does not require monotonicity. We formulate the appropriate notion of mapped coercivity in complex Banach spaces by employing the real part of the duality pairing and establish a corresponding surjectivity theorem for weakly continuous operators on reflexive complex Banach spaces. The resulting theory generalizes the existing framework for real Banach spaces and complements the classical existence theory for monotone operators in the complex setting. As applications, we prove existence of weak solutions for several nonlinear complex-valued partial differential equations, including the stationary nonlinear Schrödinger equation, the stationary complex Ginzburg–Landau equation, and the complex Maxwell problem with solution dependent coefficients. These examples demonstrate that mapped coercivity yields a unified existence theory for nonlinear operator equations in complex Banach spaces beyond the classical monotone framework.

We also apply this concept to saddle-point problems with nonlinear diffusion. Here, the challenge is to weaken the requirement of weak continuity, because this is often not achievable when the diffusion coefficient does not converge in L^∞.

References

[1] R. Becker and M. Braack, Mapped coercivity for the stationary Navier-Stokes equations and their finite element approximations. *Comput. Methods Appl. Math.* **25** (2025), no. 3, 547–560.

[2] R. Becker and M. Braack, The concept of mapped coercivity for nonlinear operators in Banach spaces. *J. Funct. Anal.* **289** (2025), no. 3, Paper no. 110893, 19 pp.

On the Söhngen Problem for the Finite Hilbert Transform

Javier Carrillo-Alanís[1], **Guillermo P. Curbera**[1], **Olvido Delgado**[2]

[1]*Facultad de Matemáticas, Universidad de Sevilla, Sevilla, Spain*

[2]*Escuela Superior de Ingeniería Informática, Universidad de Sevilla, Sevilla, Spain*

We address the problem of inversion of the finite Hilbert transform on $L^2(-1,1)$, first noted by H. Söhngen in 1954.

References

[1] S. Okada and D. Elliott, The finite Hilbert transform in L^2. Math. Nachr. **153** (1991), 43–56.

[2] H. Söhngen, Zur Theorie der endlichen Hilbert-Transformation. (German) *Math. Z.* **60** (1954), 31–51.

[3] F. G. Tricomi, *Integral Equations.* Pure and Applied Mathematics, Vol. V. Interscience Publishers, Inc., New York; Interscience Publishers Ltd., London, 1957.

From Convolutions to Polyconvolutions

Luís Castro

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Convolution methods have long provided one of the most effective bridges between nonlocal phenomena and transform techniques. Their strength lies in the fact that complicated integral expressions may become simple products, multipliers, or symbols after the appropriate transform is applied. This talk explores recent developments [1, 2] in this direction, focusing on transform-generated convolutions and polyconvolutions associated with Fourier-sine, Fourier-cosine, Hartley, and Fourier transforms.

The first part of the talk presents a three-function polyconvolution on the half-line, generated by the Fourier-sine and Fourier-cosine transforms and by an oscillatory weight. Its explicit eight-term kernel, transform factorization, boundedness properties, multilinear Young-type estimates, weighted Saitoh-type inequalities, and Titchmarsh-type consequences will be discussed. We also show how this structure leads to a Watson-type theory for even-order differential operators, including unitary criteria and explicit inverse formulae, as well as constructive solutions of Fredholm and Barbashin-type integral equations.

The second part is devoted to a multidimensional Gaussian-weighted convolution associated with the Hartley and Fourier transforms. We describe its factorization identity, mapping properties, Young and Hausdorff–Young estimates, weighted inequalities, and applications to Fredholm, Barbashin, and Cauchy-type problems.

A unifying message of the talk is that, although the physical-space kernels may appear highly intricate, their transform-side structure can be remarkably transparent. This viewpoint leads to explicit solution formulae, quantitative norm estimates, unitarity criteria, and a coherent framework for treating nonlocal integral and integro-differential equations.

References

[1] L. P. Castro, N. M. Tuan and T. Tuan, Polyconvolutions and their applications to Fredholm and Barbashin-type integral equations. *Quaest. Math.* **49** (2026), no. 3, 335–369.

[2] L. P. Castro, N. T. H. Phuong and T. Tuan, Qualitative properties and applications of convolutions associated with Hartley and Fourier transforms. *Bol. Soc. Mat. Mex. (3)* **32** (2026), no. 2, Paper no. 54, 46 pp.

A Galerkin Finite Element Method for the Fractional Calderón Problem

Mukul Dwivedi[1], **Jesse Railo**[2], **Andreas Rupp**[1]

[1] *Department of Mathematics, Saarland University, Saarbrücken, Germany*
[2] *Computational Engineering, LUT University, Lappeenranta, Finland*

We study a numerical reconstruction strategy for the potential in the fractional Calderón problem from a single partial exterior measurement. The forward model is the fractional Schrödinger equation in a bounded domain, with prescribed exterior Dirichlet datum and corresponding measurement of the exterior flux in an open observation set. Building on the theoretical foundations of the fractional Calderón problem [3], we propose a decomposition strategy and a Galerkin–Tikhonov method to recover the potential through a stabilized least-squares formulation. We establish existence, uniqueness and conditional convergence of the discrete reconstructor with a priori error estimates, and we demonstrate compatibility with practical schemes for the integral fractional Laplacian [1]. Numerical experiments in one and two dimensions show stable reconstructions of smooth and discontinuous potentials [2].

Acknowledgments

The authors thank Pu-Zhao Kow and Janne Nurminen for many helpful discussions related to this work. Dwivedi and Rupp have been supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – 577175348. Railo was supported by the Research Council of Finland through the Flagship of Advanced Mathematics for Sensing, Imaging and Modelling (grant # 359183), the Emil Aaltonen Foundation, and the Jenny and Antti Wihuri Foundation.

References

[1] G. Acosta and J. P. Borthagaray, A fractional Laplace equation: regularity of solutions and finite element approximations. *SIAM J. Numer. Anal.* **55** (2017), no. 2, 472–495; `https://doi.org/10.1137/15M103395`

[2] M. Dwivedi, J. Railo and A. Rupp, A Galerkin finite element method for the fractional Calderón problem. *Preprint* arXiv:2603.26287; `https://arxiv.org/abs/2603.26287`

[3] T. Ghosh, M. Ghosh and G. Uhlmann, The Calderón problem for the fractional Schrödinger equation. *Anal. PDE* **13** (2020), no. 2, 455–475.

Spectral Synthesis and Complexity

Alex Iosevich

University of Rochester, New York, USA

We study both Euclidean and Riemannian manifold manifestations of the following basic question: suppose that the Fourier transform of a distribution is supported k-dimensional manifold, or, more generally, on a set of positive k-dimensional packing content. If f is in L^p, where p is suitably small depending on k, when can we conclude that f is identically zero? This question has numerous applications and connections to central problems in harmonic analysis like the restriction conjecture and signal recovery.

A Class of inf-sup Stabilized Scott–Vogelius Pairs on General Shape-Regular Simplicial Grids

Volker John[1,2]

[1] *Weierstrass Institute for Applied Analysis and Stochastics (WIAS), Berlin, Germany*
[2] *Freie Universität Berlin, Department of Mathematics and Computer Science, Berlin, Germany*

Physically-consistent finite element methods for incompressible flow problems should be locally mass conserving (divergence-free) and should correctly balance bulk forces (pressure-robust).

The first part of the talk revisits these properties for the incompressible Stokes problem in the mixed finite element framework. While many stable and convergent mixed elements have been developed throughout the past five decades, most classical methods relax the divergence constraint and only enforce the condition discretely. As a result, these methods introduce a pressure-dependent consistency error which can potentially pollute the computed velocity. These methods are not pressure-robust in the sense that a contribution from the right-hand side, which influences only the pressure in the continuous equations, impacts both velocity and pressure in the discrete equations. This part of the talk is based on [1, 2].

Both properties are achieved by Scott–Vogelius pairs of finite elements. However, the also important inf-sup stability, to lead to a well-posed problem, is given only on very special types of grids. The second part of this talk presents a class of methods that extend the velocity space by a Raviart–Thomas enrichment such that the good properties of Scott–Vogelius pairs are preserved and inf-sup stability can be proved on general shape-regular simplicial grids. Using these methods does not require the computation of terms on facets of the grids. After having introduced the full method, a reduced method will be derived, where in the saddle point problem only a piecewise constant pressure is computed. Finally, a method is presented that uses divergence-free basis functions, so that even the solution of a saddle point problem is avoided. A survey on the available analytic results will be given and numerical examples demonstrate the behavior of the methods. The work was performed in collaboration with Ch. Merdon (WIAS) and X. sLi (Shandong) and it is based on the publications [3–6].

References

[1] V. John, *Finite Element Methods for Incompressible Flow Problems*. Springer Series in Computational Mathematics, 51. Springer, Cham, 2016; doi:https://doi.org/10.1007/978-3-319-45750-5

[2] V. John, A. Linke, Ch. Merdon, M. Neilan, and L. G. Rebholz, On the divergence constraint in mixed finite element methods for incompressible flows. *SIAM Rev.* **59** (2017), no. 3, 492–544; doi:https://doi.org/10.1137/15M1047696

[3] V. John, X. Li, C. Merdon and H. Rui, Inf-sup stabilized Scott–Vogelius pairs on general shape-regular simplicial grids by Raviart–Thomas enrichment. *Math. Models Methods Appl. Sci.* **34** (2024), no. 5, 919–949; https://doi.org/10.1142/S0218202524500180

[4] V. John, X. Li and C. Merdon, Pressure-robust $L^2(\Omega)$ error analysis for Raviart–Thomas enriched Scott–Vogelius pairs. *Appl. Math. Lett.* **156** (2024), Paper no. 109138, 7 pp.; https://doi.org/10.1016/j.aml.2024.109138

[5] N. Ahmed, V. John, X. Li and C. Merdon, Inf-sup stabilized Scott–Vogelius pairs on general shape-regular simplicial grids for Navier-Stokes equations. *Comput. Math. Appl.* **168** (2024), 148–161; https://doi.org/10.1016/j.camwa.2024.05.034

[6] V. John, X. Li and C. Merdon, Divergence-free decoupled finite element methods for incompressible flow problems. *J. Sci. Comput.* **108** (2026), no. 2, Paper no. 44, 34 pp.; https://doi.org/10.1007/s10915-026-03368-8

Element-Based WENO Stabilization and Convex Limiting for Finite Element Discretizations of Hyperbolic Problems

Dmitri Kuzmin

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High-order continuous and discontinuous Galerkin approximations of hyperbolic conservation laws require nonlinear stabilization near discontinuities and additional mechanisms for enforcing physically admissible states. We present an element-based framework that combines dissipation-based weighted essentially nonoscillatory (WENO) stabilization with monolithic convex limiting. Hermite-type WENO reconstructions are used only to measure local smoothness; they do not overwrite the finite element approximation. A reconstruction-based sensor activates low-order artificial viscosity near nonsmooth features and retains a high-order consistent formulation in smooth regions. Cell-vertex averaging incorporates Taylor data from common-vertex neighbors without explicitly enlarging the reconstruction stencil. This construction yields well-defined nonlinear residuals, optimal convergence for smooth solutions, and crisp nonoscillatory shock resolution [1–3].

Physical admissibility is enforced by rewriting explicit updates as convex combinations of intermediate states. For continuous Galerkin methods, the use of inexact quadrature and Rusanov-type artificial dissipation leads to a low-order scheme that is provably invariant domain preserving under a CFL condition. The corresponding convex decomposition uses intermediate cell averages. High-order accuracy is recovered by adding locally conservative element contributions. Monolithic convex limiting constrains these corrections so that the associated intermediate nodal states, and hence the final update, remain in a prescribed convex invariant set [4, 5]. The proposed limiting and stabilization framework represents an algebraic counterpart of discontinuous Galerkin strategies based on cell-average flux limiting, WENO slope correction in troubled cells, and positivity-preserving slope limiting [6]. The numerical behavior of the presented methods will be illustrated by numerical examples for scalar conservation laws and compressible flow problems.

This talk is based on joint work with Joshua Vedral, Andreas Rupp and Hennes Hajduk [1–3, 5].

References

[1] D. Kuzmin and J. Vedral, Dissipation-based WENO stabilization of high-order finite element methods for scalar conservation laws. *J. Comput. Phys.* **487** (2023), Paper no. 112153, 20 pp.

[2] J. Vedral, A. Rupp and D. Kuzmin, Strongly consistent low-dissipation WENO schemes for finite elements. *Appl. Numer. Math.* **210** (2025), 64–81.

[3] J. Vedral and D. Kuzmin, Cell-vertex WENO schemes with shock-capturing quadrature for high-order finite element discretizations of hyperbolic problems. *Comput. & Fluids* **317** (2026), Paper no. 107201.

[4] D. Kuzmin, Monolithic convex limiting for continuous finite element discretizations of hyperbolic conservation laws. *Comput. Methods Appl. Mech. Engrg.* **361** (2020), 28 pp.

[5] D. Kuzmin, H. Hajduk and J. Vedral, An element-based convex limiting framework for continuous Galerkin methods with nonlinear stabilization. *J. Comput. Phys.* **560** (2026), Paper no. 114952, 23 pp.

[6] X. Zhang and C.-W. Shu, Maximum-principle-satisfying and positivity-preserving high-order schemes for conservation laws: survey and new developments. *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.* **467** (2011), no. 2134, 2752–2776.

Convexity and Fourier Transform

Elijah Liflyand

Department of Mathematics, Bar-Ilan University, Ramat-Gan, Israel

We present a variety of problems that show how convexity type conditions imply a certain behavior of the Fourier transform. Convexity of domains will also come into play. If time allows we shall discuss how and where convexity type assumptions may be relaxed.

Olsen's Sharp Inequalities

Alexander Meskhi[1,2]

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We establish the following inequality

$$\|g \cdot I_\alpha f\|_X \le C\, \|g\|_Y\, \|f\|_Z, \quad f \in Z, \quad g \in Y, \tag{1}$$

in the sharp form, where X, Y and Z are Morrey or Morrey-type spaces defined on $\mathbb{R}^n$, and I_α is the fractional integral operator:

$$(I_\alpha f)(x) = \int\limits_{\mathbb{R}^n} \frac{f(y)}{|x-y|^{n-\alpha}}\, dy, \quad 0 < \alpha < n, \quad x \in \mathbb{R}^n.$$

Inequality (1) is sharp in the sense that the Morrey norm $\|g\|_Y$ in (1) can not be replaced by the smaller one.

We treat both diagonal and off-diagonal cases. In some cases the problem is studied from ball Banach function spaces viewpoint.

The talk is based on the papers [1-3].

References

[1] L. Grafakos and A. Meskhi, On sharp Olsen's and trace inequalities for multilinear fractional integrals. *Potential Anal.* **59** (2023), no. 3, 1039–1050; `https://doi.org/10.1007/s11118-022-09991-y`

[2] A. Meskhi and Y. Sawano, Trace and Olsen's Sharp Inequalities, *Trans. A. Razmadze Math. Inst.* **80** (2006), *Published online.*

[3] A. Meskhi and Y. Sawano, Sharp Olsen's inequality in Diagonal Case. *Submitted.*

Bounded Toeplitz Products on the Hardy Space

Ryan O'Loughlin

Department of Mathematics, University of Reading, Reading, U.K.

A classical theorem of Toeplitz states that a Toeplitz operator on the Hardy space is bounded if and only if its symbol is bounded. In 1994, Sarason asked for a corresponding characterisation of when a product of two Toeplitz operators is bounded. In this talk, I will present a solution for a class of admissible symbols, showing that boundedness of the operator product is equivalent to boundedness of the product of the symbols. I will also discuss the boundary behaviour of the symbols and the connection with the two-weight Riesz projection.

References

[1] R. O'Loughlin, Bounded Toeplitz products on the Hardy space. *Preprint* arXiv:2501.14069, 2025; `https://arxiv.org/abs/2501.14069`

Noncommutative Fujita Exponent

Michael Ruzhansky

Ghent Analysis & PDE Centre, Department of Mathematics: Analysis, Logic and Discrete Mathematics, Ghent University, Belgium

In this talk we will discuss the Fujita exponent for nonlinear heat equations in several noncommutative settings. In particular, we focus on unimodular Lie groups and Hormander suns of squares.

Numerical Ranges and their Possible Shapes: some Recent Developments

Ilya Spitkovsky

Division of Science and Mathematics, New York University Abu Dhabi (NYUAD), UAE

The numerical range (a.k.a. Hausdorff set) $W(A)$ of a bounded linear operator A acting on a Hilbert space $\mathscr{H}$ is the image of the unit sphere of $\mathscr{H}$ under the mapping $x \mapsto (Ax, x)$. This set is convex due to the classical Toeplitz–Hausdorff theorem, is an elliptical disk if $\dim \mathscr{H} = 2$ (the Elliptical Range Theorem), admits a complete description if $\dim \mathscr{H} = 3$ (see [1] or Section 6.2 of [2]), but not much more is known about its shape in the case of higher dimensions, except that when $\dim \mathscr{H} = n < \infty$ it is the convex hull of some algebraic curve $C(A)$ associated with A.

The talk consists of two parts. Part one is prompted by the work of Gau–Wang–Wu who in [3] obtained some results on the numerical ranges of so called *Foguel operators*. The latter are defined as $F_T = \begin{bmatrix} S^* & T \\ 0 & S \end{bmatrix}$, where S is the right shift on $\mathscr{H}$ while T is an arbitrary bounded linear operator acting on $\mathscr{H}$.

When $T = 0$, the Foguel operator F_0 is nothing but the bilateral shift, and so $W(F_0)$ is the open unit disk. It was suggested in [3] that $W(F_{aI})$ for non-zero $a \in \mathbb{C}$ might be an elliptical disk. We provide, based on [4], an explicit description of $W(F_{aI})$, showing in particular that it is something else.

Part two is devoted to the finite dimensional case when A is *reciprocal*. This means that in the orthonormal basis with respect to which the matrix of A has a constant main diagonal (which can always be arranged), is is actually tridiagonal, and the off diagonal entries satisfy $a_{i+1,i}a_{i,i+1} = 1$, $i = 1, \dots n-1$. Conditions are established for $W(A)$ to have elliptical shape and, moreover, for $C(A)$ to contain an elliptical component or even to consist only of ellipses. The respective results are contained in [5]–[9].

Acknowledgments

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References

[1] D. S. Keeler, L. Rodman and I. M. Spitkovsky, The numerical range of 3×3 matrices. *Linear Algebra Appl.* **252** (1997), 115–139.

[2] P. Y. Wu and H.-L. Gau, *Numerical Ranges of Hilbert Space Operators*. Encyclopedia of Mathematics and its Applications, 179. Cambridge University Press, Cambridge, 2021.

[3] H.-L. Gau, K.-Zh. Wang and P. Y. Wu, Numerical ranges of Foguel operators. *Linear Algebra Appl.* **610** (2021), 766–784.

[4] M. Jiang and I. M. Spitkovsky, Numerical ranges of some Foguel operators. *Oper. Matrices* **17** (2023), no. 1, 179–185.

[5] N. Bebiano, J. da Providência, I. Spitkovsky and K. Vazquez, Kippenhahn curves of some tridiagonal matrices. *Filomat* **35** (2021), no. 9, 3047–3061.

[6] T. Geryba and I. M. Spitkovsky, On some 4-by-4 matrices with bi-elliptical numerical ranges. *Linear Multilinear Algebra* **69** (2021), no. 5, 855–870.

[7] M. Jiang and I. M. Spitkovsky, On some reciprocal matrices with elliptical components of their Kippenhahn curves. *Spec. Matrices* **10** (2022), 117–130.

[8] M. Jiang and I. M. Spitkovsky, Unified approach to reciprocal matrices with Kippenhahn curves containing elliptical components. *Linear Multilinear Algebra* **73** (2025), no. 7, 1346–1368.

[9] I. M. Spitkovsky and J. Zhou, Work in progress.

Dual of Hardy Inequality

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For $0 < p \leq 1$, the classical Hardy inequality states that

$$\Big(\sum_{n=1}^{\infty} |\widehat{f}_n|^p n^{p-2}\Big)^{\frac{1}{p}} \lesssim \|f\|_{H^p}.$$

In this talk I discuss its dual.

References

[1] S. Elaydi, *An Introduction to Difference Equations*. Undergraduate Texts in Mathematics. Springer, New York, 2005; https://doi.org/10.1007/0-387-27602-5

[3] E. Emmrich, *Discrete Versions of Gronwall's Lemma and Their Application to the Numerical Analysis of Parabolic Problems*. TU Berlin, Fachbereich Mathematik, Preprint no. 637, 1999; https://www3.math.tu-berlin.de/preprints/files/Preprint-637-1999.pdf

Geometric Hardy Inequalities on the Heisenberg Group

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We present Hardy inequalities on the Heisenberg group in a form reminiscent of Davies' inequalities in the Euclidean setting. Our results have two notable features. First, they improve previously known inequalities, yielding sharper estimates. Second, we prove them without imposing any boundary regularity assumptions. We also discuss how these Davies-type Hardy inequalities can be converted into sub-Laplacian eigenvalue lower bounds. This talk is based on recent joint work with Rupert L. Frank (LMU Munich and Caltech) and Ari Laptev (Imperial College London).

Analytic Pseudo-Differential Calculus Via the Bargmann Transform

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In the 70th, Berezin used the Bargmann transform to translate problems in operator theory into an analytic pseudo-differential calculus, the so-called Wick calculus, where corresponding operators map suitable classes of entire functions into other classes of entire functions. In the same manner, the Toeplitz operators correspond to so-called anti-Wick operators on the Bargmann transformed side.

In the talk we find convenient characterizations of Wick and anti-Wick operators acting on the Bargmann images of so-called Pilipović spaces (extended class of classical Fourier invariant Gelfand–Shilov spaces). We show links between global ellipticity in the real pseudo-differential calculus and the Wick calculus, as well as links between Wick and anti-Wick operators.

References

[1] N. Teofanov and J. Toft, Pseudo-differential calculus in a Bargmann setting. *Ann. Acad. Sci. Fenn. Math.* **45** (2020), no. 1, 227–257.

[2] N. Teofanov, J. Toft and P. Wahlberg, Pseudo-differential operators with isotropic symbols, Wick and anti-Wick operators, and hypoellipticity. *J. Math. Pures Appl. (9)* **167** (2022), 48–100.

[3] J. Toft, Images of function and distribution spaces under the Bargmann transform. *J. Pseudo-Differ. Oper. Appl.* **8** (2017), no. 1, 83–139.

[4] J. Toft, Wick and anti-Wick characterizations of linear operators on spaces of power series expansions. *J. Fourier Anal. Appl.* **28** (2022), no. 5, Paper no. 71, 65 pp.

Critical Curves for Heat Equations with Convolution-Type Nonlinearities

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This work investigates the heat equations with nonlocal power-law nonlinearities involving a Riesz potential. We introduce the Fujita-type critical exponent, which characterizes the global behavior of solutions: global existence for small initial data in supercritical case, and finite-time blow-up critical and subcritical cases. It is remarkable that the critical Fujita exponent is not determined by the usual scaling argument, but instead arises in an unconventional manner, similar to the results of Cazenave et al. [1] for the heat equation with a time-nonlocal nonlinearity. The result on global existence, provides a positive answer to the hypothesis proposed by Mitidieri and Pohozaev in [2]. We further establish global nonexistence results for the above heat equation, where the Riesz potential term is replaced by a more general convolution operator, thereby extending the Mitidieri–Pohozaev's results established in the aforementioned work. Proofs of the blow-up results are obtained using a nonlinear capacity method specifically adapted to the structure of the problem, while global existence is established via a fixed-point argument combined with the Hardy–Littlewood–Sobolev inequality. The main results are a joint work with Ahmad Z. Fino [3, 4].

References

[1] T. Cazenave, F. Dickstein and F. B. Weissler, An equation whose Fujita critical exponent is not given by scaling. *Nonlinear Anal.* **68** (2008), no. 4, 862–874.

[2] È. Mitidieri and S. I. Pokhozhaev, Liouville theorems for some classes of nonlinear nonlocal problems. (Russian) *Tr. Mat. Inst. Steklova* **248** (2005), Issled. po Teor. Funkts. i Differ. Uravn., 164–184 ISBN: 5-02-033713-7 *Proc. Steklov Inst. Math.* **2005**, no. 1(248), 158–178

[3] A. Z. Fino and B. T. Torebek, Parabolic problems whose Fujita critical exponent is not given by scaling. *Calc. Var. Partial Differential Equations* **65** (2026), no. 4, Paper no. 116, 24 pp.

[4] A. Z. Fino and B. T. Torebek, Fujita-type results for parabolic equations with Hartree-type nonlinearities. *Nonlinear Anal.* **273** (2026), Paper no. 114228.

Schrödinger Operator on a Star Graph with Nondefinite Weight Function

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We investigate the indefinite Laplacian on a star graph G with $n = n_+ + n_-$ edges, where the differential operator acts as minus second derivative on n_+ edges and plus second derivative on n_- edges. The operator is subject to Dirichlet conditions at the outer vertices and Kirchhoff conditions at the central vertex. We establish that this operator is similar to a self-adjoint operator in the Hilbert space $L^2(G)$ and that its eigenfunctions constitute a Riesz basis. Additionally, we provide a complete characterization of its point spectrum.

Abstracts of Contributed Talks

Dynamical Mixed Boundary-transmission Problem of Thermo-Electro-Magneto-Elastic Composite Body Containing an Interface Crack

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We study a three-dimensional dynamical problem for a composite body consisting of two adjacent, anisotropic, thermo-electro-magneto-elastic components sharing a common interface and governed by the Green–Lindsay model of generalized thermo-electro-magneto-elasticity [1] with distinct material properties. The composite structure contains an interfacial crack, and the interaction between the adjacent components is modeled by a fully coupled system of dynamic partial differential equations alongside corresponding mixed initial-boundary-transmission conditions. By applying the Laplace transform, the dynamical problem is reduced to an elliptic problem for pseudo-oscillation equations containing a complex parameter τ. Using potential theory and pseudodifferential equations on manifolds with boundaries, we establish existence and uniqueness theorems in appropriate Sobolev spaces for any complex parameter τ satisfying $\operatorname{Re}\tau > 0$. Furthermore, asymptotic estimates for the solution and its derivatives are derived for sufficiently large $|\tau|$, building upon similar estimate techniques established for heterogeneous structures under mixed models: Green–Lindsay's model of generalized thermo-electro-magneto-elasticity in one component and Green–Lindsay"s model of generalized thermo-elasticity in the other one (see [2, 3]). Finally, we prove that the inverse Laplace transform of the pseudo-oscillation solution yields the unique solution to the original dynamical problem within the designated function spaces.

References

[1] M. Aouadi, Some theorems in the generalized theory of thermo-magnetoelectroelasticity under Green–Lindsay's model. *Acta Mech.* **200** (2008), 25–43; `https://doi.org/10.1007/s00707-007-0576-z`

[2] T. Buchukuri, O. Chkadua and D. Natroshvili, Mixed boundary-transmission problems of the generalized thermo-electro-magneto-elasticity theory for piecewise homogeneous composed structures. *Trans. A. Razmadze Math. Inst.* **175** (2021), no. 2, 163–198.

[3] T. Buchukuri, O. Chkadua and D. Natroshvili, Mixed- and crack-type dynamical problems of electro-magneto-elasticity theory. *Georgian Math. J.* **28** (2021), no. 4, 533–553.

Dobrushin Coefficient and Uniform Ergodicity of Operator Semigroups

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The asymptotic behavior of physical and probabilistic systems constitutes a fundamental area of research at the intersection of mathematics and physics. Because the dynamics of these systems are typically governed by one-parameter operator semigroups (such as C_0-semigroups), analyzing their long-term behavior reduces to studying the limiting properties of these operators. Investigating these properties within an abstract framework is highly advantageous, providing a unified approach that seamlessly encompasses both classical and quantum settings.

The primary goal of this talk is to explore the asymptotic stability of C_0-semigroups defined on abstract state spaces by employing a Dobrushin ergodicity coefficient. Additionally, we investigate the rate of convergence when the semigroup exhibits uniform ergodicity.

Enhanced Long-Time Existence for the Whitham–Boussinesq System with Surface Tension in One and Two Dimensions

Madina Kamet, Achenef Tesfahun

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We study the fully dispersive Whitham–Boussinesq system for surface water waves with surface tension (Bond number $\beta = 1$) in one and two horizontal dimensions, in the shallow-water scaling with amplitude parameter $\epsilon \in (0, 1]$ and shallowness parameter $\mu \in (0, 1]$. The dispersion enters through the multiplier

$$K_\mu(\xi) = \left(1 + \mu|\xi|^2\right) \frac{\tanh\left(\sqrt{\mu}\,|\xi|\right)}{\sqrt{\mu}\,|\xi|}\,, \tag{1}$$

whose capillary factor makes the symbol grow at short waves, in sharp contrast with the decaying multiplier of the pure-gravity models of [2].

For initial data of size A_0 in the μ-weighted energy space $X^s_\mu(\mathbb{R}^d)$ with $s > 11/4$, satisfying a non-cavitation condition (and irrotationality when $d = 2$), we prove existence, uniqueness and uniform bounds on the time scales

$$T_{\epsilon,\mu} \simeq \mu^{1/3}\epsilon^{-4/3}A_0^{-4/3} \quad (d = 1), \qquad T_{\epsilon,\mu} \simeq \mu^{1/6}\epsilon^{-4/3}A_0^{-4/3}\big(\log(2 + \epsilon^{-1})\big)^{-1/3} \quad (d = 2), \tag{2}$$

improving the amplitude exponent of the classical energy-method lifespan ϵ^{-1} [1] to $\epsilon^{-4/3}$, with an explicit shallowness gain.

The system is diagonalised into a pair of half-wave groups $e^{\mp itm_\mu(D)}$, $m_\mu = |\xi|K_\mu^{1/2}$. Frequency-localised dispersive bounds with decay $|t|^{-1/2}$ $(d = 1)$ and $|t|^{-1}$ $(d = 2)$ are upgraded by TT^* arguments to Strichartz estimates with the admissible pair $(4, \infty)$ in one dimension and, in two dimensions, with $q = 2^+$. Combined with a modified energy method whose key new ingredient is a μ-uniform weighted Kato-Ponce commutator estimate adapted to the growing symbol (1), these close the bootstrap; the threshold $s > 11/4$ is produced by the heaviest quantity $K_\mu\nabla\cdot v$ and is sharp within the scheme.

This research was funded by the Science Committee of the Ministry of Science and Higher Education of Kazakhstan (Grant No. AP26194665) and by the Nazarbayev University Research Grants Program (Grant No. SSH2025017)

References

[1] M. O. Paulsen, Long time well-posedness of Whitham–Boussinesq systems. *Nonlinearity* **35** (2022), no. 12, 6284–6348; `https://arxiv.org/abs/2203.13336`

[2] D. Pilod, S. Selberg, N. S. Taki and A. Tesfahun, Improved existence time for the Whitham equation and a Whitham–Boussinesq system. Preprint arXiv:2508.08809; `https://arxiv.org/abs/2508.08809`

Existence and Uniqueness of Solutions to Non-Local Nonlinear Reaction-Diffusion Equations

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We study nonlinear reaction-diffusion equations with non-local source terms, in which the results arise in population dynamics and epidemiology. Under mild conditions on the initial data, interaction kernel, and nonlinearity, we prove the existence, uniqueness, and positivity of bounded classical solutions. The analysis employs a Hammerstein–Volterra nonlinear integral formulation and a constructive monotone iteration scheme that preserves positivity and continuity. Special cases with vanishing reproduction rate and constant source intensity on finite time intervals are also investigated. The results extend known one-dimensional models to a general multidimensional framework.

Solvability of a System of Nonlinear Wave Equations in Three-Dimensional Space

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We consider a general system of N coupled semilinear wave equations in $\mathbb{R}^3$ with possibly different propagation speeds. The nonlinear terms are assumed to be positive, monotone and subject to a suitable concavity condition. Such systems arise naturally in the mathematical description of interacting wave phenomena and provide a flexible framework for modelling nonlinear interactions between several components.

Our approach is based on reducing the Cauchy problem to an equivalent nonlinear integral system through Duhamel's representation formula. We then construct a sequence of successive approximations and establish its monotone and uniform convergence by combining integral estimates with structural properties of the nonlinearities. This yields a constructive procedure for approximating solutions and allows us to derive quantitative convergence estimates.

As a consequence, we prove the existence of positive bounded classical solutions on $\mathbb{R}^3 \times (0, T]$ for arbitrary finite $T > 0$. In addition, we establish uniqueness within a natural class of admissible solutions satisfying suitable comparison bounds. The obtained results extend earlier solvability results for scalar nonlinear wave equations to systems with an arbitrary number of interacting components and multiple propagation speeds.

Several examples are presented to illustrate the applicability of the theory and the range of nonlinear interactions covered by the assumptions.

References

[1] Kh. A. Khachatryan and M. Mikaeili Nia, Solvability of a system of nonlinear wave equations in three-dimensional space. *p-Adic Numbers Ultrametric Anal. Appl.* **18** (2026), no. 3, 271–287.

Failure Envelope and Poisson's Effect Equations for Composite Anisogrid Laminates

Aleksandr Oleinikov, Tatiana Kuzmina

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Multi-directionally reinforced laminates of carbon fibres with a polymer matrix offer a unique combination of properties that makes them very attractive for many applications. Analysing such laminates requires finding the constant coefficients of a system of differential and/or integral equations. The number of these constants is proportional to the complexity of the constitutive equations that close this system, such as the law of elasticity and the failure criterion.

The purpose of this report is to present a new approach to coupled meso- and macro-level direct description of failure of anisogrid composite plies and laminates, which radically reduces the number of coefficients. Failure envelopes are proposed as the minimum inner envelope in stress space, which defines the ultimate envelope (the ply with the first fibre failure) for all ply orientations. This envelope would be easy to use to predict failure.

Definition A laminate whose elastic moduli in the direction perpendicular to the fibres and shear moduli in the fibre planes are negligible compared to the elastic modulus in the direction of the fibres will be called *anisogrid* [1].

The following Theorem 1 presents the main result of this report and establishes the ultimate envelope and Poisson's effect at failure for the laminates.

Theorem 1 *Let the anisogrid multidirectional laminate be symmetrical and balanced, and be in a plane stress state under the action shear loading τ_α applied to the laminate at plane with normal, which at an angle α to the direction of the fibres in ply-0.*

Then

(a) *the failure stress $\tau_{\varphi_l^*}(\alpha)$ of such laminate is determined as [1]*

$$\tau_{\varphi_l^*}(\alpha) = \min_{\varphi_l}\{X_{\varphi_l} F_{\varphi_l}(a, \overline{h}_{\varphi_l})\}, \tag{1}$$

where X_{φ_l} is the longitudinal strength of the φ_l layer material, $\overline{h}_{\varphi_l} = h_{\varphi_l}/H$ is the relative thickness of the φ_l-layer, H is the laminate thickness, and F_{φ_l} is the coefficient proportional to which the stress τ_α is distributed within the layers of this laminate.

(b) *if the Poisson's ratio of the laminate satisfies the inequality*

$$\frac{|X_{\varphi_l}^-|}{X_{\varphi_l}^+} \le \nu_{0,90}^L \le \sqrt{\frac{E_0^L}{E_{90}^L}}\,, \tag{2}$$

then under uniaxial tension of the laminate along the fibre ply-0, the ply-90 fails.

Note The constitutive equation (1) of the mathematical model of laminate failure is limited by only two physical constants – the longitudinal strengths of the layers under tension and compression.

For example, for an isogrid laminate, $\overline{h}_{\varphi_l} = h = idem$ with the plies stacking $(0_n, 45_n, 90_n, 135_n)$ inequality (2) can be written in simple form

$$\frac{X^+_{\varphi_l}}{|X^-_{\varphi_l}|} \geq 3 \tag{3}$$

and graph (1) has the form

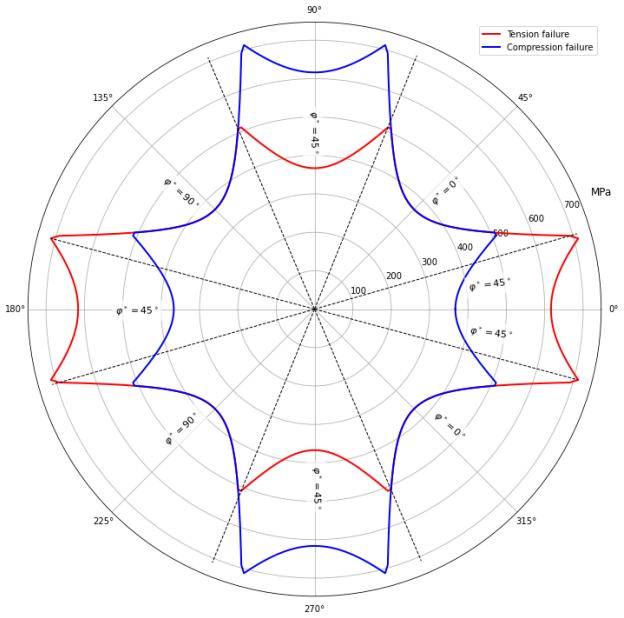

Figure 1: Failure envelope (1) for an isogrid laminate carbon/epoxy Hexcel IM7/8552 under shear loading τ_α

In conclusion, some consequences and applications of Theorem 1 are given.

References

[1] T. Kuzmina and A. Oleinikov, Two-scale failure problems of the anisogrid composite laminates. *Conference Proceedings 2nd International Conference on Modern Problems of Mathematics, Mechanics and Their Applications (MPMMA 2026), Baku*, 2026, pp. 932–937; `https://doi.org./10.58225/mpmma.2026.932`

Dispersive Estimates for Full Dispersion KP Equations

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We derive some dispersive estimates for the full dispersion Kadomtsev–Petviashvili (KP) equation introduced by David Lannes to overcome some shortcomings of the classical KP equations. The proof of these estimates combines the stationary phase method with sharp asymptotics on asymmetric Bessel functions. These estimates are used to prove local well-posedness of the Cauchy problem at a low regular Sobolev space.

References

[1] D. Pilod, J.-C. Saut, S. Selberg and A. Tesfahun, Dispersive estimates for full dispersion KP equations. *J. Math. Fluid Mech.* **23** (2021), no. 1, Paper no. 25, 24 pp.

Rational Approximation of Unitary Operator Groups and Applications to Hyperbolic Equations

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The present study is devoted to the approximation of the unitary operator group arising from the Cauchy problem for abstract hyperbolic equations; see [1]. Its main contribution is a stable rational approximation that achieves fourth-order accuracy for sufficiently regular data and also yields convergence estimates under weaker regularity assumptions.

The approximation is used to construct a two-level time-discretization method for inhomogeneous linear and semilinear problems. At each time step, the method approximates both the unknown function and its time derivative. For both classes of problems, error estimates are established under appropriate regularity assumptions on the prescribed data. The resulting method provides a unified and stable framework for the high-order time discretization of a broad class of second-order evolution problems.

References

[1] J. Rogava and Z. Vashakidze, On the approximation of the unitary operator group associated with a rotation matrix and its applications to abstract hyperbolic equations. *Preprint* arXiv:2607.21293, 2026; `https://arxiv.org/abs/2607.21293`

Orthogonal Wavelet Bases on the Generalized Vilenkin Groups

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The well known classical Vilenkin group V_p, $p = 2, 3, \ldots$, is associated with the cyclic group of order p. Similar structures, associated with an arbitrary finite abelian group G instead of a cyclic group (called generalized Vilenkin groups), were recently introduced jointly with A. Vodolazov in [1]. Such a group is also associated with a dilation matrix M whose set of digits is isomorphic to G. Wavelet systems on the generalized Vilenkin groups were studied jointly with M. Babushkin. An algorithmic method for the construction of orthogonal wavelet bases is presented (see [2]). These bases consist of compactly supported test functions (i.e. functions whose Fourier transform is also compactly supported).

References

[1] A. M. Vodolazov and M. A. Skopina, Generalized Vilenkin groups. (Russian) *Mat. Zametki* **116** (2024), no. 4, 489–503; translation in *Math. Notes* **116** (2024), no. 3-4, 588–599.

[2] M. Babushkin and M. Skopina, Orthogonal Wavelet Bases on Generalized Vilenkin Groups. *Lobachevskii J. Math.* **47** (2026), no. 3, 991–1000.

Some New Sharp Boundedness Result Concerning Linear functionals on Banach Spaces

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The convergence and summability of Fourier series with respect to classical orthonormal systems (ONS) – such as the trigonometric, Haar, and Walsh systems – are well-established in mathematical analysis, particularly for differentiable functions. Due to the favorable localization and structural properties of these classical systems, regular functions yield well-behaved Fourier series. However, for general ONS, the behavior of Fourier expansions is markedly different and highly irregular. As demonstrated by a classical result of Banach [2], Fourier series with respect to certain ONS may fail to be summable even for constant functions.

In this paper, we bridge this gap by investigating the Cesàro summability of order $\alpha > 0$ of Fourier series with respect to general ONS for the class of functions C_L, defined as functions whose derivatives belong to the Lipschitz class Lip_1. Treating Lip_1 as a Banach space, we analyze a sequence of linear functionals $B_n(f, x)$ and introduce an auxiliary sequence of functions $H_n(x)$ constructed from the kernel functions $Q_n(u, x)$. The primary contribution of this work is the establishment of precise, pointwise boundedness criteria for the Cesàro sums $\sigma_n^{\alpha}(x, f)$ on a subset of full measure $G \subset [0, 1]$. We prove that the condition $H_n(x) = O(1)$ is sufficient to guarantee the boundedness of Cesàro sums at any point $x \in G$ for all functions in C_L. Crucially, we show that this result is sharp: if $\limsup_{n\to\infty} |H_n(x_0)| = +\infty$ at some point $x_0 \in G$, there exists a function whose Cesàro sums diverge at that point. Finally, we present an existence theorem proving the existence of an ONS and a function $g \in C_L$ for which the corresponding Cesàro sums diverge almost everywhere on $[0, 1]$. These results are shown to be the best possible in a special sense, offering new insights into the summability of smooth functions under general orthonormal expansions.

Acknowledgments

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References

[1] G. Alexits, *Convergence Problems of Orthogonal Series*. International Series of Monographs in Pure and Applied Mathematics, Vol. 20. Pergamon Press, New York–Oxford–Paris, 1961.

[2] S. Banach, Sur la divergence des séries orthogonales. (French) *Studia Math.* **9** (1940), 139–155.

[3] L. Gogoladze and V. Tsagareishvili, Fourier coefficients of continuous functions. (Russian) *Mat. Zametki* **91** (2012), no. 5, 691–703; translation in *Math. Notes* **91** (2012), no. 5-6, 645–656.

[4] B. S. Kashin and A. A. Saakyan, *Orthogonal Series*. Translations of Mathematical Monographs, 75. American Mathematical Society, Providence, RI, 1989.

[5] V. Tsagareishvili and G. Tutberidze, Some problems of convergence of general Fourier series. (Russian) *Izv. Nats. Akad. Nauk Armenii Mat.* **57** (2022), no. 6, 70–80; translation in *J. Contemp. Math. Anal.* **57** (2022), no. 6, 369–379.

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Useful Visitor Information for TAPDE 2026

Annotated quick-reference for workshop participants

Keep this guide handy while travelling in Tbilisi. It summarizes the practical points from the TAPDE 2026 visitor-information email. For the most current operational details, use the linked official or service websites.

Airport taxi guidance

- Street taxis in Tbilisi are typically **unmetered**, so tourists can occasionally be overcharged.
- Prefer a ride-hailing service such as Bolt.
- A Bolt journey between Tbilisi International Airport and the city center should cost about **EUR 15-16**.
- Police often do not allow Bolt taxis to stop at the **arrivals terminal**; this restriction does not normally apply at the **departures terminal**.
- Check the driver's live location in the app: a driver may still wait at arrivals even when your pick-up point is set to departures.

Note: Practical takeaway: if airport pick-up is difficult, confirm the terminal and meeting point in the app before walking outside.

Temporary road restrictions in central Tbilisi

Liberty Square and Rustaveli Avenue are undergoing rehabilitation work. These areas are expected to be accessible to pedestrians only, with vehicular traffic restricted.

Note: Allow extra travel time and expect taxi or bus routes around the city center to change.

Public transport in Tbilisi

- Pay with **Visa, MasterCard, or Amex**; **Google Pay and Apple Pay** can also be used.
- **Tap your card or device immediately after boarding** a bus or minibus.
- Travelling without a valid ticket may result in a **fine from ticket inspectors**.
- Plan routes with Google Maps or the **Tbilisi Transport** app: Android / iOS.
- Public transport information: **ttc.com.ge/en**.

One-week public transport pass

A one-week travel pass can be purchased at metro stations. Look for a Metro Service Plus (MS+) kiosk inside the station.

Georgian Railway – Tbilisi / Batumi

For TAPDE 2026 participants who will also attend the XVI Annual International Conference of the Georgian Mathematical Union:

- Tbilisi to Batumi: August 30, 2026
- Batumi to Tbilisi: September 5 or 6, 2026
- Recommended booking platform: **tre.ge/en**

The Georgian Railway booking platform is also available as a mobile app on the App Store and Google Play.

Note: Book early if your onward travel depends on these dates.

Lunch and food delivery

- Nearby lunch suggestions: **Samikitno-Matchakhela** and **Khinkali Factory**.
- Workshop-area address: **77 Merab Kostava St., Tbilisi 0171**.
- Delivery services include **Glovo, Wolt, and Bolt Food**.
- During summer, the cafeteria on the second floor of the University of Georgia **may be closed**.

Mobile data and eSIM

To ensure you have seamless cellular data upon arrival in Georgia, we highly recommend purchasing an **eSIM in advance from abroad** through the official **MyMagti app or website**.

- For a fee of **10 GEL**, you can select a Georgian number and choose a prepaid mobile plan.
- Complete the mandatory remote legal registration by securely uploading your **passport photo** on MyMagti.
- After purchase, connect to Wi-Fi and scan the provided **QR code** to install the eSIM profile.
- If you wish to activate and use the line **before passing through Georgian border control**, make sure **roaming is enabled** in your device settings.

Note: Official MyMagti access: magticom.ge/mymagti.

Cards, cash, and currency exchange

Most restaurants, supermarkets, and other establishments accept **Visa, MasterCard, and Amex** cards.
If you prefer cash, consider exchange offices on **Pekini Avenue**, near the conference venue.
Banks, airports, and tourist areas such as Liberty Square may offer less favourable exchange rates.

Access to the University of Georgia

Construction near the University of Georgia means there are currently only **two possible routes** to the conference venue. The shortest route begins from **Saakadze Descent**, near the **Rompetrol gas station**, and continues to the **University of Georgia (UG)** on Merab Kostava Street.

Open the Rompetrol to University of Georgia route in Google Maps

Note: Email clients may block embedded maps, so the direct route link is the reliable fallback.

Nearby hotel suggestions

If you have not yet booked accommodation, these options are within walking distance of the workshop venue:

Holiday Inn Tbilisi	1 Twentysix May Square, Tbilisi 0171
Nina Palace Hotel	80/82 Merab Kostava St., Tbilisi

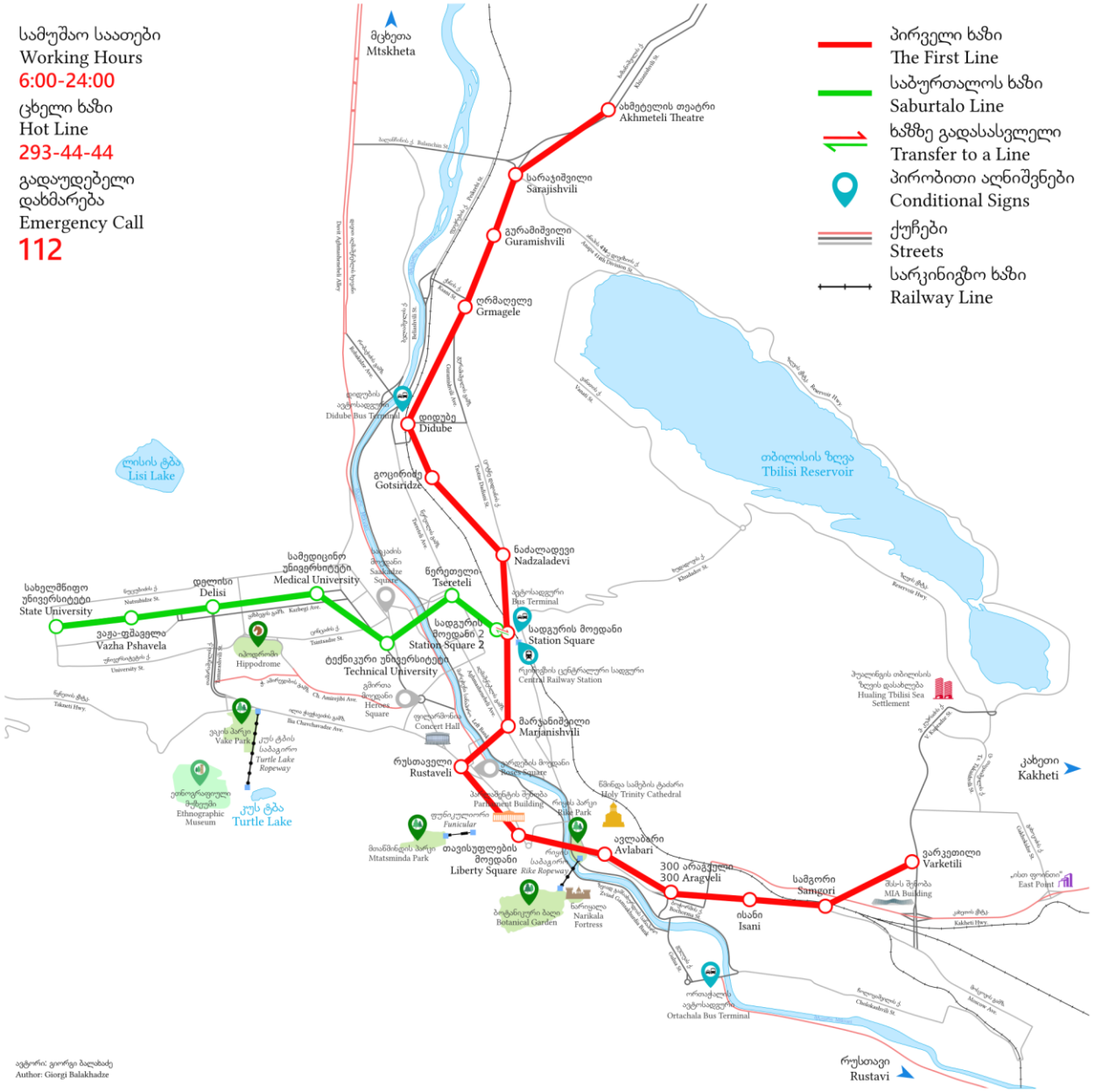
Useful links at a glance

Bolt	bolt.eu/en/cities/tbilisi
Tbilisi public transport	ttc.com.ge/en
Georgian Railway	tre.ge/en
MyMagti eSIM	magticom.ge/mymagti
Venue walking route	https://maps.app.goo.gl/HQMiaMKwsbucPBBK8

TAPDE 2026 Organizing Committee

This document is a practical summary of the visitor-information email and is intended as a quick reference.

The closest underground station to the conference venue is the **Technical University** station. See the map below.



In the event of an **emergency**, please call **112**.

Partner Institutions and Sponsors

The **TAPDE2025** workshop is part of a project that has received support from The University of Georgia (UG) in Tbilisi.

Sponsor

https://tapde-workshop.ug.edu.ge/

Prepared by R. Duduchava with the help of ChatGPT